2026 WAMC 8 Answer Key

1	**D**	6	**C**	11	**E**	16	**D**	21	**E**
2	**B**	7	**C**	12	**A**	17	**C**	22	**D**
3	**B**	8	**C**	13	**B**	18	**E**	23	**A**
4	**D**	9	**C**	14	**A**	19	**D**	24	**D**
5	**A**	10	**A**	15	**E**	20	**B**	25	**C**

(Answer key is for reference; full solutions begin on the next page.)

2026 WAMC 8 Solutions

1. What is the value of $1111000 + 111100 + 11110 + 1111$?

   (A) $123,321$ **(B)** $1,223,321$ **(C)** $1,234,311$ **(D)** $1,234,321$ **(E)** $1,234,567$

   Proposed by Ryan Tang

   Solution. Align the numbers by place value and add vertically:

   $$\begin{array}{r} 1111000 \\ 111100 \\ 11110 \\ 1111 \\ \hline 1234321 \end{array}$$

   Therefore, the value of the sum is $\boxed{\textbf{(D)}\ 1,234,321}$.

2. Ryan writes 3 math problems for WAMO every 2 days. If he needs to write a 10-problem contest, a 25-problem contest, and a 1-problem contest, how many days will it take him to write all the problems?

   (A) 12 **(B)** 24 **(C)** 30 **(D)** 32 **(E)** 36

   Proposed by Ryan Tang

   Solution. Ryan needs to write a total of $10 + 25 + 1 = 36$ problems. He writes 3 problems every 2 days, which is a rate of $\frac{3}{2}$ problems per day. At this rate, the total number of days required is $36 \div \frac{3}{2} = \boxed{\textbf{(B)}\ 24}$.

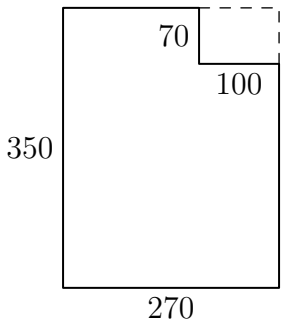
3. While writing problems, Ryan uses a spinner with three labeled regions, G, A, and N, to choose which type of problem to write next. Region G has twice the area of region A, and region A has twice the area of region N. What is the probability that the spinner lands on region A?

   (A) $\frac{1}{7}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{4}{7}$

   Proposed by Kaiyuan Mao

   Solution. Let the area of region N be 1. Then region A has area 2, and region G has area 4. The total area of the spinner is $1+2+4 = 7$. The probability that the spinner lands on region A is therefore $\boxed{\textbf{(B)}\ \frac{2}{7}}$.

4. A simplified map of Utah is approximated as a rectangle with a smaller rectangle cut out as shown below. All lengths are in miles. What is the area of this figure?

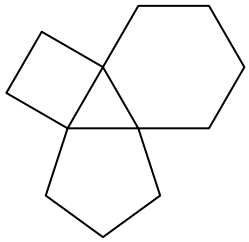
   

(A) $59,500$ **(B)** $75,600$ **(C)** $85,000$ **(D)** $87,500$ **(E)** $93,800$

Proposed by Wesley Wu

Solution. The full rectangle has width 270 miles and height 350 miles, giving area $270 \cdot 350$. The cut-out rectangle has width 100 miles and height 70 miles, giving area $100 \cdot 70$. Subtracting, the area of the figure is $270 \cdot 350 - 70 \cdot 100 = \boxed{\textbf{(D)}\ 87,500}$ square miles.

5. A square, a regular pentagon, and a regular hexagon are constructed externally on the three sides of an equilateral triangle so that each polygon shares exactly one full side with the triangle. If the triangle has side length 6, what is the perimeter of the resulting figure shown below?



(A) 72 **(B)** 84 **(C)** 90 **(D)** 96 **(E)** 108

Proposed by Vivaan Mukherjee

Solution. Each attached regular polygon has side length 6 because it shares a full side with the equilateral triangle, and that shared side is not part of the outer perimeter. Thus the contribution to the perimeter is $(4-1) \cdot 6$ from the square, $(5-1) \cdot 6$ from the pentagon, and $(6-1) \cdot 6$ from the hexagon. The total perimeter is $3 \cdot 6 + 4 \cdot 6 + 5 \cdot 6 = 18 + 24 + 30 = \boxed{\textbf{(A)}\ 72}$.

6. Jason learns that the Six-Day War lasted from the beginning of June 5, 1967, to the end of June 10, 1967. Assuming the war progressed uniformly over time, on which day was the war 40% complete?

(A) June 5 **(B)** June 6 **(C)** June 7 **(D)** June 8 **(E)** June 9

Proposed by Ryan Tang

Solution. The war lasted from the beginning of June 5 to the end of June 10, a total of 6 days. Forty percent of 6 days is $0.4 \times 6 = 2.4$ days. Two full days after June 5 brings the time to the beginning of June 7, and the additional 0.4 day still falls on June 7. Therefore, the war was 40% complete on $\boxed{\textbf{(C)}\ \text{June } 7}$.

7. Wesley defines a rule that takes a number x and transforms it to $\frac{x+1}{x-1}$. Starting with 2, he applies the rule three times. What number does he obtain?

(A) 2 **(B)** $\frac{5}{2}$ **(C)** 3 **(D)** 4 **(E)** 5

Proposed by Wesley Wu

Solution. Define $f(x) = \frac{x+1}{x-1}$. Starting with 2, the first application gives $f(2) = \frac{3}{1} = 3$, the second gives $f(3) = \frac{4}{2} = 2$, and the third gives $f(2) = 3$. Therefore, the final number obtained is $\boxed{\textbf{(C)}\ 3}$.

8. James is interested in palindromes, which are numbers that read the same forwards and backwards, such as 121 or 434. How many positive three-digit palindromes have a digit sum equal to 8?

(A) 1 **(B)** 2 **(C)** 4 **(D)** 7 **(E)** 8

Proposed by Kaiyuan Mao

Solution. A three-digit palindrome has the form $\overline{aba}$, where $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 9\}$. Its digit sum is $2a + b$. Setting $2a + b = 8$, the valid solutions are $(a, b) = (1, 6), (2, 4), (3, 2), (4, 0)$. Thus there are $\boxed{\textbf{(C)}\ 4}$ such palindromes.

9. Let $a \circ b$ be the larger of the positive integers a and b, and let $a \square b$ be the smaller. If

$$(a \circ b) + (a \square b) = 5,$$

how many ordered pairs (a, b) of positive integers satisfy this condition?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Proposed by Alexander Yang

Solution. For any positive integers a and b, $(a \circ b) + (a \square b)$ is simply $a + b$. Thus the condition becomes $a + b = 5$. The positive integer solutions are $(1, 4), (2, 3), (3, 2), (4, 1)$, for a total of $\boxed{\textbf{(C)}\ 4}$ ordered pairs.

10. Patrick sketches a square with two opposite vertices at $(-2, 4)$ and $(3, 7)$. This square has the same area as a right triangle with one leg of length 4. What is the length of the other leg of the triangle?

(A) $\frac{17}{2}$ **(B)** 9 **(C)** $\frac{17\sqrt{2}}{2}$ **(D)** 17 **(E)** $17\sqrt{2}$

Proposed by Wesley Wu

Solution. The distance between the opposite vertices is

$$\sqrt{(3 - (-2))^2 + (7 - 4)^2} = \sqrt{5^2 + 3^2} = \sqrt{34},$$

which is the diagonal of the square. The side length is hence $\frac{\sqrt{34}}{\sqrt{2}} = \sqrt{17}$, so the area of the square is 17. For the right triangle, the area is $\frac{1}{2} \cdot 4 \cdot x = 2x$. Setting this equal to 17 gives $2x = 17$, so $x = \frac{17}{2}$. Thus, the length of the other leg is $\boxed{\textbf{(A)}\ \frac{17}{2}}$.

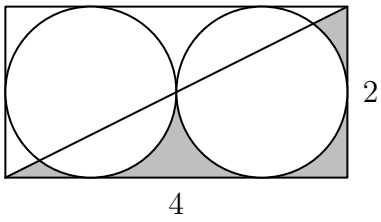
11. Vivaan is covering a 4×6 rectangular floor with 1×1 square tiles. Each tile is either blue or white, and any two tiles that share a side must be different colors. The tile in the upper-left corner is blue. If a blue tile costs $\$2$ and a white tile costs $\$3$, what is the total cost to cover the floor?

(A) 45 **(B)** 48 **(C)** 50 **(D)** 54 **(E)** 60

Proposed by James Yang

Solution. The coloring must be a checkerboard pattern. Since $4 \cdot 6 = 24$ is even, there are 12 blue tiles and 12 white tiles. The total cost is $12 \cdot 2 + 12 \cdot 3 = 24 + 36 = \boxed{\textbf{(E)}\ 60}$ dollars.

12. A 4×2 rectangle contains two congruent circles, each tangent to the top and bottom of the rectangle and tangent to each other. A diagonal is drawn from the lower-left corner to the upper-right corner. Find the area of the shaded region shown.



(A) $4-\pi$ **(B)** $8-2\pi$ **(C)** 3 **(D)** π **(E)** 2π

Proposed by Kaiyuan Mao

Solution. The two circles each have radius 1, so their total area is 2π, and the rectangle has area $4 \cdot 2 = 8$. By symmetry about the vertical line through the circles' tangency point, the diagonal splits the region outside the circles into two congruent parts, so the shaded area is half of $8-2\pi$. Therefore the shaded area is $\boxed{\textbf{(A)}\ 4-\pi}$.

13. Laura notices several small numbers written on a whiteboard and starts looking for patterns among them. She notices that they are four distinct numbers from 1 through 9 such that exactly two are prime, exactly two are perfect squares, exactly two are even, and exactly two are multiples of 3. What is the sum of the four numbers?

(A) 15 **(B)** 18 **(C)** 20 **(D)** 21 **(E)** 22

Proposed by Vivaan Mukherjee

Solution. From 1 to 9:

- Squares: $\{1, 4, 9\}$
- Primes: $\{2, 3, 5, 7\}$
- Evens: $\{2, 4, 6, 8\}$
- Multiples of 3: $\{3, 6, 9\}$

We first observe that there are no numbers satisfying more than 2 conditions. Therefore, each of the 4 numbers must satisfy 2 conditions. Therefore, we can eliminate all of the numbers that satisfy fewer than 2 constraints, which are 1, 5, 7, and 8. Our new list looks like this:

- Squares: $\{4, 9\}$
- Primes: $\{2, 3\}$
- Evens: $\{2, 4, 6\}$
- Multiples of 3: $\{3, 6, 9\}$

From our first two conditions, we see that there are 4 distinct numbers satisfying them, and therefore, 2, 3, 4, and 9 are the 4 numbers in this set. We can confirm that they work by checking the last two conditions. Adding up these 4 numbers gives us $\boxed{\textbf{(B)}\ 18}$.

14. How many ordered quadruples (a, b, c, d) of one-digit positive integers satisfy

$$\frac{ab}{cd} = \frac{a}{c} \div \frac{b}{d},$$

where $ab = a \cdot b$ and $cd = c \cdot d$?

(A) 729 **(B)** 810 **(C)** 900 **(D)** $1,000$ **(E)** $1,024$

Proposed by Ryan Tang

Solution. The condition simplifies to

$$\frac{ab}{cd} = \frac{ad}{bc} \Rightarrow b^2 = d^2.$$

Since the digits are positive, $b = d$. There are 9 choices for each of a, c, b, so the number of ordered quadruples is $9^3 = \boxed{\textbf{(A)}\ 729}$.

15. After the AMC 10, four students Alex, Kaiyuan, Jackson, and Kevin each make a statement about their scores. Assume no two students received the same score.

- Alex says, "I got the highest score."
- Kaiyuan says, "I did not get the lowest score."
- Jackson says, "I did worse than Alex, but I did not get the lowest score."
- Kevin says, "I got the lowest score."

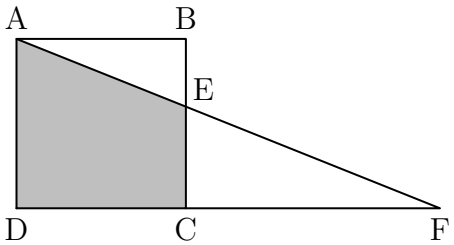
Exactly one of these statements is false. Determine the order of Alex, Kaiyuan, Jackson, and Kevin from highest score to lowest score.

(A) Alex, Kaiyuan, Jackson, Kevin **(B)** Kaiyuan, Jackson, Alex, Kevin
(C) Kaiyuan, Alex, Kevin, Jackson **(D)** Alex, Jackson, Kaiyuan, Kevin
(E) Kaiyuan, Alex, Jackson, Kevin

Proposed by Kaiyuan Mao

Solution. If Kevin were not lowest, then either Kaiyuan's or Jackson's statement would also be false, giving at least two false statements. Hence Kevin is lowest, and both Kevin and Kaiyuan tell the truth. To keep exactly one false statement, Alex must be lying, so Alex is not highest. Jackson's statement then requires Alex to be above Jackson. Thus the only possible order is $\boxed{\textbf{(E)}\ \text{Kaiyuan, Alex, Jackson, Kevin}}$.

16. Square $ABCD$ has side length 8. Point E lies on segment BC. Line AE intersects line DC at F. If $DF = 20$, find the area of quadrilateral $ADCE$.



(A) 28.8 **(B)** 35.2 **(C)** 50 **(D)** 51.2 **(E)** 57.6

Proposed by Alexander Yang

Solution. Since $ABCD$ has side length 8, $CD = 8$, and $CF = DF - DC = 20 - 8 = 12$. Also, since AD and CE are parallel, $\triangle ADF \sim \triangle ECF$ with a ratio of $\frac{DF}{CF} = \frac{20}{12} = \frac{5}{3}$. Thus, $\frac{CE}{AD} = \frac{CF}{DF} = \frac{3}{5}$, so $\frac{CE}{8} = \frac{3}{5}$, meaning that $CE = \frac{24}{5}$. Note that $ADCE$ is a trapezoid, and the area of a trapezoid the the average of the bases times the height which is $\frac{8+\frac{24}{5}}{2} \cdot 8 = \boxed{\textbf{(D)}\ 51.2}$.

17. The only valid element symbols are Ga, As, Sg, and S. No symbol has more than two letters, and neither G nor A by itself is a valid symbol. Consider the string GASGASGAS. You may group consecutive letters of the string to form element symbols, ignoring capitalization (for example, GA can represent Ga). In how many different ways can the string be broken into valid element symbols?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Proposed by Wesley Wu

Solution. Note that the first two letters GA must be the element Ga, since there is no other possible way. Now, the next element can either be Sg or S. Let us proceed with casework.

Case one: The element is Sg. If this is the case, then we must have the next element to be Ag, forcing Ga and S. Thus, there is only one valid sequence here.

Case two: The element is S. Then, we must have Ga, and once again either S or Sg. If it is S, then we must have Ga, S. If it is Sg, then we must have As. So, there are two valid sequences here.

Thus, in total we have $2 + 1 = \boxed{\textbf{(C)}\ 3}$ valid sequences.

18. A 5-digit password formed using only the digits 1 and 2. It contains more 2s than 1s, is divisible by 3, and is not divisible by 4. What is the sum of all such passwords?

(A) $22,212$ **(B)** $55,556$ **(C)** $55,566$ **(D)** $77,778$ **(E)** $77,787$

Proposed by Kaiyuan Mao

Solution. We proceed with casework. We first note that in a 5-digit integer with fewer 1s than 2s and no other digits, there must be either 3 2s, 4 2s, or 5 2s. We also note that this password must be divisible by 3, which means that the sum of the digits is a multiple of 3 as well. If there were 3 2s, then the digit sum becomes 8, 4 2s yields a digit sum of 9, and 5 2s yields 10. The only case here that works is having 4 2s and a 1. The only numbers that satisfy this are
12222
21222
22122
22212
22221
Among these remaining numbers, the password cannot be divisible by 4. A password is divisible by 4 if and only if the last two digits of the number are divisible by 4. Checking the remaining passwords, we find that 22212 is the only password among these divisible by 4 because 12 is divisible by 4. Adding up the remaining passwords yield a final answer of $\boxed{\textbf{(E)}\ 77,787}$.

19. Bruce is picking oranges from a tree. Exactly 1% of oranges on the tree are blue, and the rest are orange. The first orange Bruce picks is blue. If the probability that the second orange he picks is blue is $\frac{1}{111}$, how many oranges were on the tree at the start?

(A) 500 **(B)** 700 **(C)** 800 **(D)** $1,000$ **(E)** $1,300$

Proposed by Ryan Tang

Solution. Let there be x blue oranges on the tree before Bruce picks the first one. Then, there are $100x$ total oranges. After Bruce picks the first orange, there are now $x-1$ blue oranges and $100x-1$ total oranges. So the probability the second orange he picks is blue is $\frac{x-1}{100x-1} = \frac{1}{111}$.
Cross multiplication gives $111(x-1) = 100x-1$, so $111x-111 = 100x-1$ and $11x = 110$, so $x = 10$.
As a result, there were $100x = \boxed{\textbf{(D)}\ 1,000}$ oranges on the tree at the start.

20. A crystal is formed by attaching two congruent square pyramids to the two square faces of a square prism. The side length of the square base of the prism is 3, and a space diagonal of the prism has length $3\sqrt{6}$. The distance between the two pyramid apexes is 12. What is the volume of the crystal?

(A) 63 **(B)** 72 **(C)** 81 **(D)** 90 **(E)** 108

Proposed by Vivaan Mukherjee

Solution. To find the volume of this crystal, we split the crystal up into the square prism and the two square pyramids. We first note that the volume of each of the pyramids can be represented by $\frac{s^2h}{3}$, where s is the side length of the square base, and h is the height of the pyramid. We also note that the prism shares the same base as both pyramids and its height is the sum of the heights of the pyramids subtracted from 12. Therefore, the volume of the prism can be represented as $s^2(12-2h)$.

We have the side length of the square base, 3, and we wish to find the height of either the prism or the pyramids. We are given the information that the space diagonal of the prism is $3\sqrt{6}$. The diagonal of a square is $\sqrt{2}$ multiplied by the side length of the square, so $3\sqrt{2}$. Using the Pythagorean theorem on the plane perpendicular to the square base, where we have

$$\begin{aligned}(3\sqrt{6})^2-(3\sqrt{2})^2&=(12-2h)^2\\54-18&=(12-2h)^2\\36&=(12-2h)^2\\12-2h&=6\\h&=3\end{aligned}$$

Plugging in our values for s and h into our volume formulas yields that the volume of one pyramid is $\frac{3^2(3)}{3}=9$, so the volume of both pyramids is 18. Additionally, the volume of the prism is $3^2(12-2(3))=3^2(6)=54$. Therefore, the total volume of the crystal is $18+54=\boxed{\textbf{(B)}\ 72}$.

21. Let n be the least positive integer such that n has exactly 10 positive divisors and $2n$ has exactly 15 positive divisors. How many positive divisors does n^2 have?

(A) 18 **(B)** 20 **(C)** 24 **(D)** 25 **(E)** 27

Proposed by James Yang

Solution. Notice that if n has 10 divisors, it must be in the form pq^4, or in the form x^9 where p, q and x are distinct primes. Then, we have a few cases to consider.

Case one: $p=2$. Then, $2n$ is in the form p^2q^4, which does indeed have 15 divisors. The least positive integer satisfying this form is $n=2\cdot 3^4=162$.

Case two: $q=2$. Then, $2n=pq^5$, which has 12 divisors, invalid.

Case three: $p,q\neq 2$. Then, $2n=2pq^4$, which has 20 divisors, invalid.

Case four: $n=x^9$. Then, if $x=2$, then we get $2n=2^{10}$ which has 11 factors. If $x\neq 2$, then $2n=2x^9$ which has 20 factors. Either way, it isn't valid.

So, the smallest n satisfying these conditions is $n=162$. $n^2=2^2\cdot 3^8$, which has $3\cdot 9=\boxed{\textbf{(E)}\ 27}$ divisors.

22. How many integers of the form a^a satisfy

$$2^9<a^a<2^{25},$$

where a is a positive integer?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Proposed by Ryan Tang

Solution. Note that the sequence is exclusively increasing, we simply just need to find the largest and smallest a satisfying this condition, and all integers in that range will satisfy the condition. Since a is bounded by power of two, we are motivated to test powers of 2 for a. First, lets consider $a = 4$. Then, $a^a = (2^2)^4 = 2^8$. So, 4 is too low, just by a bit. Next, lets try 5. We know that $5^5 > 4^5 = (2^2)^5 = 2^{10}$, so 5 is valid. Now, for our max bound let us consider $a = 8$, another power of 2. In this case, $8^8 = (2^3)^8 = 2^{24}$, which is still below 2^{25}. Let's consider $a = 9$. $9^9 > 8^9 = (2^3)^9 = 2^{27}$, which is too large. Thus, our valid range is 5 to 8, inclusive, which consists of $\boxed{\textbf{(D)}\ 4}$ numbers.

23. A number is called *diabolical* if it contains the consecutive digits 14 or 34. For example, 9341 and 1434 are diabolical, but 3574 is not. How many four-digit diabolical numbers are there?

(A) 556 **(B)** 560 **(C)** 576 **(D)** 596 **(E)** 600

Proposed by Vivaan Mukherjee

Solution. We proceed with the Principle of Inclusion-Exclusion. We first calculate the number of 4-digit integers that include instances of 14, then add the number of 4-digit integers that include instances of 34, the subtract the number of 4-digit integers that include instances of both 14 and 34.

There are 3 cases to deal with for the first part of this solution. If 14 is in the first 2 digits of this number, there are 10 ways to choose the third digit, and 10 ways to choose the final digit, yielding 100 numbers. If 14 is not located in the first 2 digits of this number, there are 9 ways to choose the digit in the first position (anything but 0) and 10 ways to choose the digit in the remaining position. This yields a total of $100 + 2 \cdot 90 = 280$ numbers. However, this manner of counting counts the number 1414 twice, because of its two instances of 14, so we need to subtract 1 from our total, giving us 279 numbers.

For the second part of this solution, we can realize that this case is identical to the first case. Therefore, there will be another 279 numbers found here, and adding that total to the first case yields 558 4-digit integers.

To finish this problem, we subtract the number of 4-digit integers with both 14 and 34. There are only 2 numbers in this case - 1434 and 3414. Thus, we subtract 2 from our running total to give us a final answer of $\boxed{\textbf{(A)}\ 556}$ 4-digit integers.

24. A three-digit integer n has the following property:

- when 1 is subtracted from n, the result is divisible by 2;
- when 3 is subtracted from n, the result is divisible by 4;
- when 5 is subtracted from n, the result is divisible by 6;
- $\ldots$
- when an odd integer k is subtracted from n, the result is divisible by $k + 1$.

Find the value of k for which there is exactly one possible value of n.

(A) 7 **(B)** 9 **(C)** 11 **(D)** 13 **(E)** 15

Proposed by Vivaan Mukherjee

Solution. The condition says that for each odd $m \le k$, $n - m$ is divisible by $m + 1$. Equivalently,

$$n \equiv m \equiv -1 \pmod{m+1}.$$

So $n \equiv -1 \pmod{L}$, where

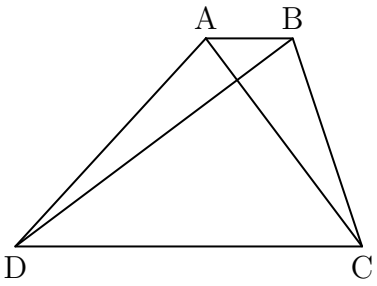
$$L = \operatorname{lcm}(2, 4, 6, \ldots, k+1).$$

Thus $n + 1$ is a multiple of L between 101 and 100. We want exactly one such multiple. Check choices:

- $k = 13$: $L = \operatorname{lcm}(2, 4, 6, 8, 10, 12, 14) = 840$, and the only multiple in $[101, 1000]$ is 840. So $n = 840 - 1 = 839$ is unique.
- $k = 15$: L includes 16, so $L = 1680 > 1000$ gives no solution.
- Smaller k give much smaller L, hence many multiples.

Therefore $\boxed{\textbf{(D)}\ 13}$.

25. In trapezoid $ABCD$, $AB \parallel CD$. Suppose the diagonals AC and BD have lengths 3 and 4, respectively, and $AB + CD = 5$. Find the area of trapezoid $ABCD$.



(A) $\frac{9}{2}$ **(B)** 5 **(C)** 6 **(D)** $\frac{15}{2}$ **(E)** 10

Proposed by Kaiyuan Mao

Solution. Translate AB downwards such that A corresponds with point C.
Let the point B is sent to be called B'.
Because of translations, $AC = BB' = 3$. Also, note that because $AB \parallel CD$, DCB' are collinear. So $DB' = DC + CB' = DC + AB = 5$. Because $DB = 4$, get that $\triangle DBC$ is a $3 - 4 - 5$ triangle.
So the height of the trapezoid is $\frac{3 \cdot 4}{5} = \frac{12}{5}$ and the area is $\frac{1}{2} \cdot 5 \cdot \frac{12}{5} = \boxed{\textbf{(C)}\ 6}$.

Alternatively, drop altitudes from A and B down to CD and set up a system.